

# MA 2000: Introduction to Formal Mathematics (Fall 2010)

## Exam 2 (take-home portion)

NAME:

### Instructions

This portion of Exam 2 is worth 40 points. On the next page, you will see four theorems and you are to prove any three of them. Each of the three proofs that you complete is worth 10 points for total of 30 points. **Your written presentation of the proofs (which includes spelling, grammar, punctuation, clarity, and legibility) is worth the remaining 10 points.**

I expect your solutions to be *well-written, neat, and organized*. You should write in *complete sentences* when appropriate. Do not turn in rough drafts. What you turn in should be the “polished” version of potentially several drafts. Feel free to type up your final version.

The L<sup>A</sup>T<sub>E</sub>X source file of this exam is also available if you are interested in typing up your solutions using L<sup>A</sup>T<sub>E</sub>X. I’ll be happy to help you do this.

The simple rules for this portion of the exam are:

1. You may freely use any theorems that we have discussed in class or that you have previously proven in homework, but you should make it clear where you are using a previous result and which result you are using. For example, if a sentence in your proof follows from the Fundamental Theorem of Arithmetic, then you should say so.
2. You are NOT allowed to use results that we have not covered, unless you prove them.
3. You are NOT allowed to copy someone else’s work.
4. You are NOT allowed to let someone else copy your work.
5. You are allowed to discuss the problems with each other and critique each other’s work.

This portion of Exam 2 is due by 9:30AM on Tuesday, November 9. You should turn in this cover page and all of the proofs you have decided to submit.

To convince me that you have read and understand the instructions, sign in the box below.

Signature:

Good luck and have fun!

Write the final version of your proofs on separate paper. You do not need to turn in this page, but I would like you to turn in the cover page.

**Theorem 1.** If  $x$  is a rational number and  $y$  is an irrational number, then  $x + y$  is irrational.

**Theorem 2.** Let  $x, y, z \in \mathbb{N}$ , where  $x$  is prime. Then  $x$  divides  $yz$  if and only if  $x$  divides  $y$  or  $x$  divides  $z$ .

**Theorem 3.** Let  $p_1, p_2, \dots, p_n$  be  $n$  distinct points arranged on a circle. Then the number of line segments joining all pairs of points is  $(n^2 - n)/2$ .\*

**Theorem 4.** Let  $a_1 = 2, a_2 = 4$ , and  $a_{n+2} = 5a_{n+1} - 6a_n$  for all  $n \geq 1$ . Then  $a_n = 2^n$  for all  $n \in \mathbb{N}$ .

---

\*Here is a picture of what things look like when  $n = 5$ .

